

## Interference Estimation for Multiuser Systems

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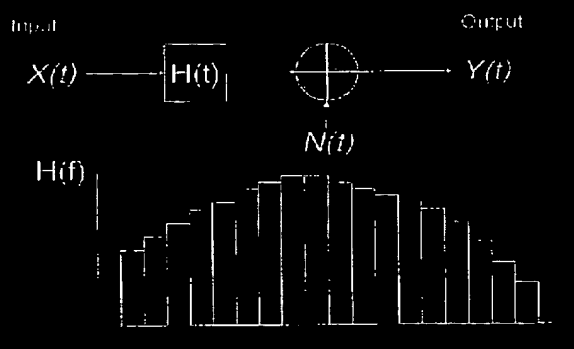
### Outline

- Aggregate Interference
- Interference Coupling Functions for Multiuser Detection
  - Block
  - Recursive
- Conclusions

### Multicarrier for Detecting Aggregate Interference

- Transmitter Optimization
  - Increase data rate and/or line reach
  - Reduce power consumption
- Track the changing environment conditions
  - React faster
  - Prevent modem retraining

### Multicarrier Transmission



- Modulation done in frequency domain
- Transmit signal is IDFT of subchannel symbols
- Robust to frequency selective environments

## Transmit Optimization-Waterfilling

Problem:

Solution: If

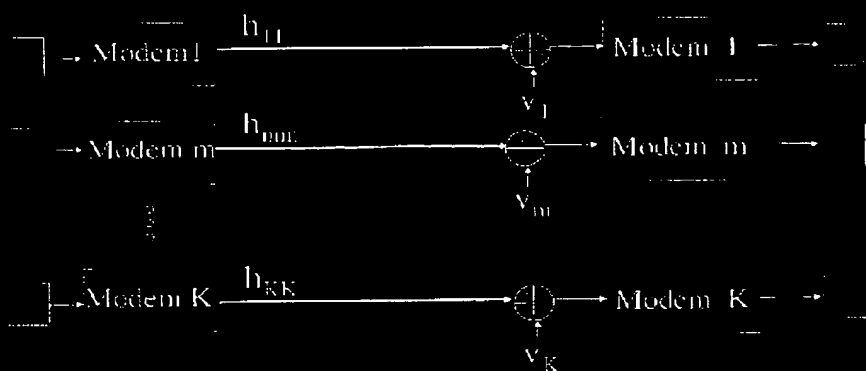
, then pour energy in trough using

$$\frac{1}{g(f)}$$

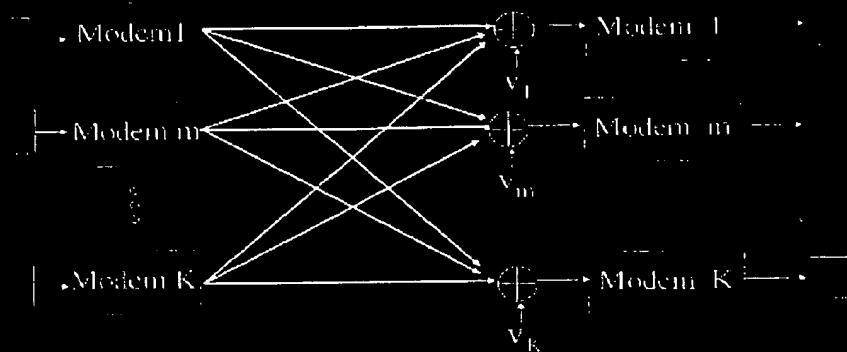
$\lambda$



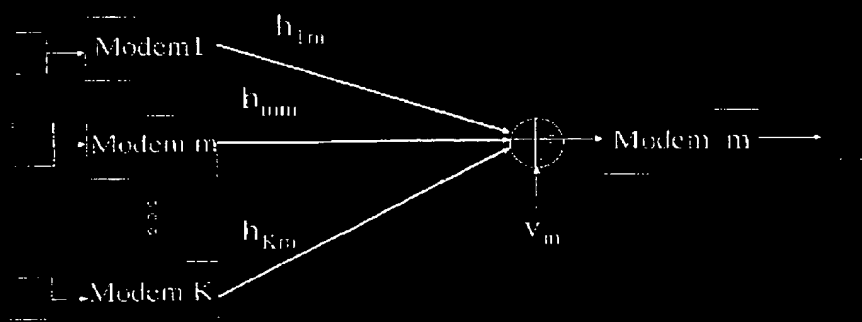
## No Crosstalk



## Crosstalk



## Crosstalk



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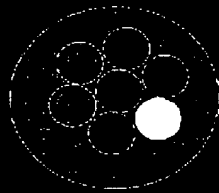
## The Main Technology Issue



Cross-section of cable

- Crosstalk – interference between lines  
*Largest source of impairment and performance loss*

## Profile 1



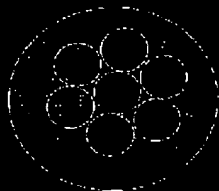
Cross-section of cable

RED=OFF  
ISDN ON  
ADSL ON  
HDSL ON

Profile: State determined by the number and the kind of crosstalkers present.

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## Profile 2



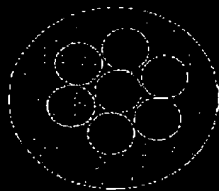
Cross-section of cable

RED=OFF  
ISDN ON  
ADSL OFF  
HDSL ON

Profile: State determined by the number and the kind of crosstalkers present.

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## Profile 3



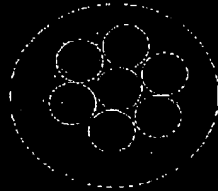
Cross-section of cable

RED=OFF  
ISDN ON  
ADSL OFF  
HDSL ON  
T1 ON

Profile: State determined by the number and the kind of crosstalkers present.

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## Profile d



Cross-section of cable

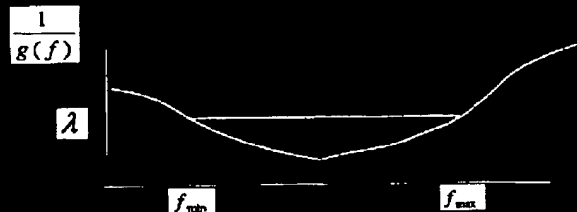
RED-OF-  
ISDN ON  
ADSL OFF  
HDSL ON  
T1 ON  
G. LITE ON

Profile: State determined by the number and the kind of crosstalkers present.

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## Profile

- Use profiles to track changing crosstalk conditions.
- Noise measurements matched to a set of profiles.
- Each profile has an optimized bit and energy table associated with it.



## Crosstalk Environments

- NEXT and FEXT (Near-End and Far-End Crosstalk)
- Different services, e.g, HDSL, ADSL, ISDN

US Transmitter  
Modem i

NEXT

DS Receiver  
Modem m

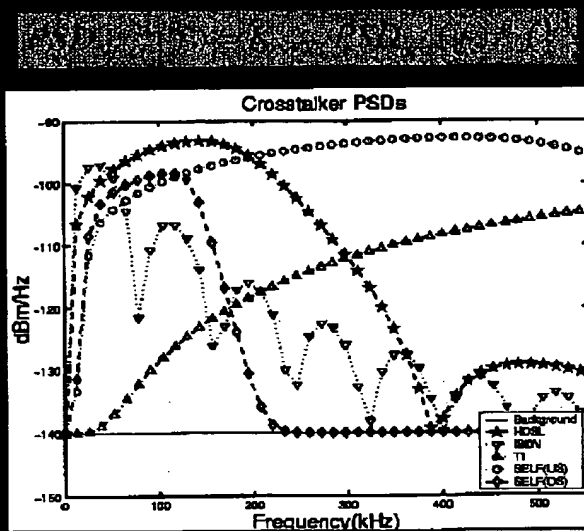


FEXT

US Receiver  
Modem m

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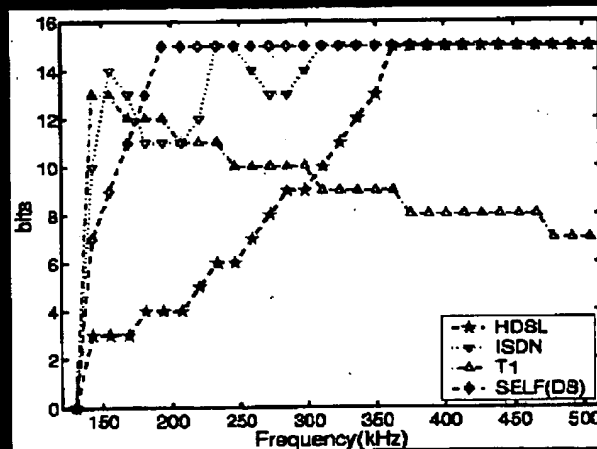
## NEXT Crosstalk Example



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## Dk Tables Example for Downstream (DS)



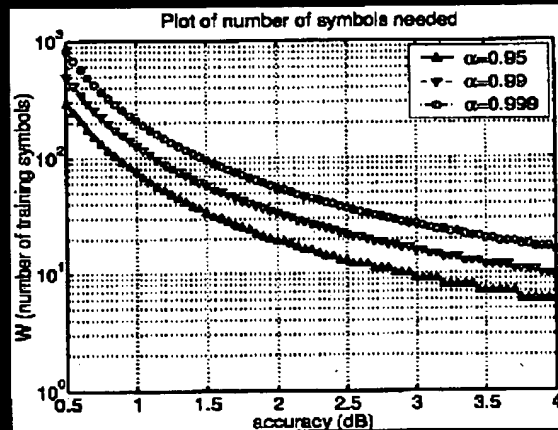
## Accumulating a Profile

- Assume block stationarity
- For each tone  $i$ , estimate the crosstalk variance over a window of length  $W$ :



## Number of Symbols Needed

Find smallest  $W$  such that:



## MAP Profile Detection

- Given a set of stored noise profiles,  $\{P_i\}$ , minimize the probability of choosing an incorrect profile.
- Choose the profile that maximizes the a posteriori probability  $P_i$ , where  $P_i$  has Gaussian pdf.
- Assuming a window of  $L$  multicarrier symbols, MAP algorithm becomes

$$\hat{P}_i = \arg \max_i \left( \frac{1}{L} \sum_{l=1}^L \frac{1}{\sigma_i^2} \left| \sum_{k=1}^K \hat{S}_{k,l} \right|^2 \right)$$

where

$$\hat{S}_{k,l} = \frac{1}{L} \sum_{l=1}^L S_{k,l}$$

## A Simple ML Example with $L=1$ , $P=3$

noise power measurements are  $[N_1^2, N_2^2, N_3^2] = [100, 200, 300]$

j	6 PROFILES			$\sum_{l=1}^3 \ln(\sigma_{j,l}^2) + \frac{N_l^2}{\sigma_{j,l}^2}$
	$\sigma_{j,1}^2$	$\sigma_{j,2}^2$	$\sigma_{j,3}^2$	
1.	.1	.2	.3	
2.	1	2	3	
3.	10	20	30	
4.	50	100	150	
5.	200	400	600	
6.	1000	2000	3000	

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## A Simple ML Example with $L=1$ , $P=3$

noise power measurements are  $[N_1^2, N_2^2, N_3^2] = [100, 200, 300]$

j	6 PROFILES			$\sum_{l=1}^3 \ln(\sigma_{j,l}^2) + \frac{N_l^2}{\sigma_{j,l}^2}$
	$\sigma_{j,1}^2$	$\sigma_{j,2}^2$	$\sigma_{j,3}^2$	
1.	.1	.2	.3	$-5.1 + 3000 = 2994.9$
2.	1	2	3	$1.8 + 300 = 301.8$
3.	10	20	30	$8.7 + 30 = 38.7$
4.	50	100	150	$13.5 + 6 = 19.5$
5.	200	400	600	$17.7 + 1.5 = 19.2$
6.	1000	2000	3000	$22.5 + .6 = 23.1$

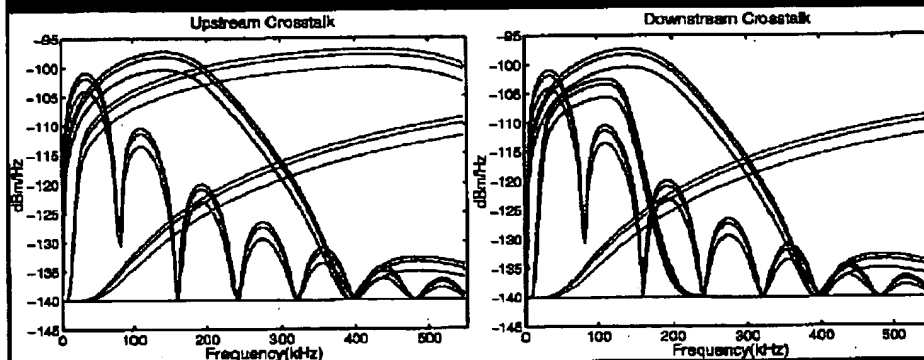
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## Simulation Setting- ADSL (G.Lite)

- 128 subchannels (32 for US, 96 for DS)
- Symbol period  $T=250$  us
- 13 profiles for both upstream and downstream stored
  - 3 ISDN (3, 7, 10 users)
  - 3 HDSL (3, 7, 10 users)
  - 3 T1 (3, 7, 10 users)
  - 3 EC ADSL using G.Lite (3, 7, 10 users)
  - Background Noise (-140dBm/Hz)

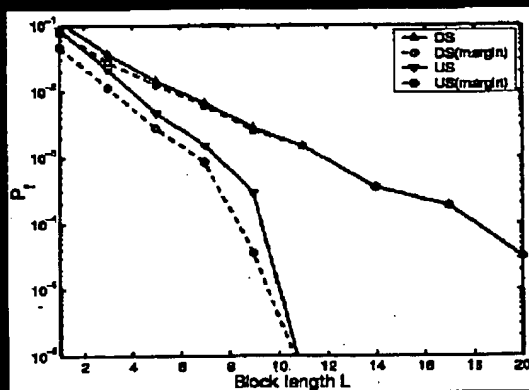
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## Profiles Used In Simulation



## Simulation Results

- Probability of choosing an incorrect profile
- Probability of the chosen bit and energy table not meeting margin



- For this scenario, use of profiles allows for doubling of the data rate.

## Advantages of Profiles

- ▣ Increase bit rates by having more profiles  
Can have fast access to crosstalk activity—no need to transmit to worst case noise scenario
- ▣ Prevent modem failure by increasing block length L
- ▣ Low complexity



R is no. of profiles, T is the symbol period, P is number of tones.

When R=16, L=20, P=128, complexity = 2.3 MIPS

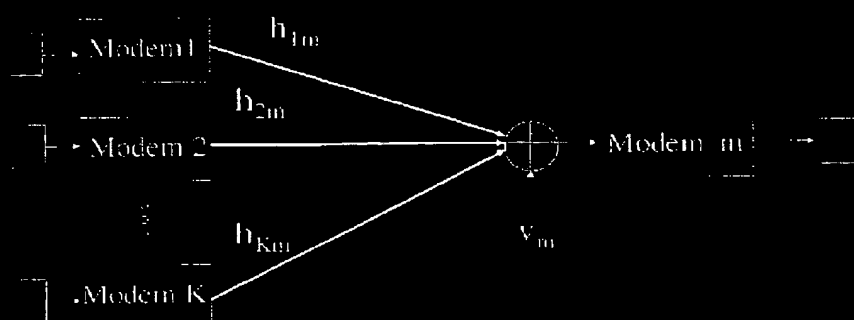
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## Crosstalk



Goal: Find  $h_{lm}$

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## Motivation

### ■ Multiuser Channel ID

Reliable channel and/or noise variance estimates for multiuser detection

Maintenance and diagnosis

Bandwidth efficient transmission

Desire to track the changing environment conditions

Estimates can be used for optimizing the transmitter

### ■ Expectation Maximization (EM)

Reduce training overhead to practically 0

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## Previous Work on Channel Estimation

### ■ EM

Introduced (Dempster, Laird, 77)

SAGE (Fessler, Hero, 94)

SISO (Kaleh, Vallet, 94)

Recursive SISO (Zamiri-Jafarian, 97)

### ■ MISO

Gaussian inputs (Foster, Weinstein, 88)

CDMA system (Bhashyam, Aazhang, 00)

### ■ Finite Alphabet

MIMO system (Talwar, 96)

SISO OFDM (Zhou, Giannakis, 01)

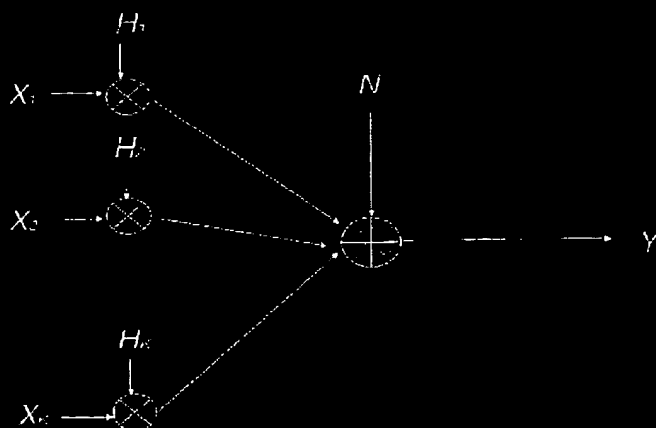
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## System Model

- Multiple access channel:  $K$  inputs, 1 output
- Modems are synchronized with same symbol period  $T$
- Receiver knows the constellations of the transmitters.
- Channel and noise are block stationary

## System Model with $K$ Tx's

In multicarrier systems with  $K$  users, each subchannel appears





## System Model

A block of received data can be collected to form:



$$Y = XH + N$$

Problem:

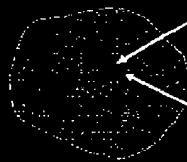
$$\text{Find } X \text{ given } Y, H, \text{ and } N$$

## Incomplete and Complete Data

Incomplete Data  $Y$

$$\max_{X \in \mathcal{X}} \log p(Y|X)$$

HARD!!!

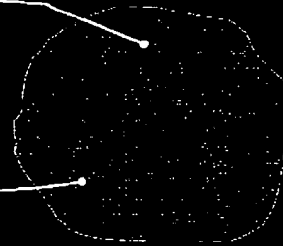


$$p(Y|X) = \prod_{i=1}^N p(y_i|x_i)$$

Complete Data  $Z=(X,Y)$

$$\max_{X \in \mathcal{X}} \log p(Z|X)$$

MUCH EASIER



## Initialization of EM

- By means of training, one can find estimates:

$$\hat{H} = \frac{1}{N} \sum_{n=1}^N \hat{X}_n \hat{X}_n^H$$

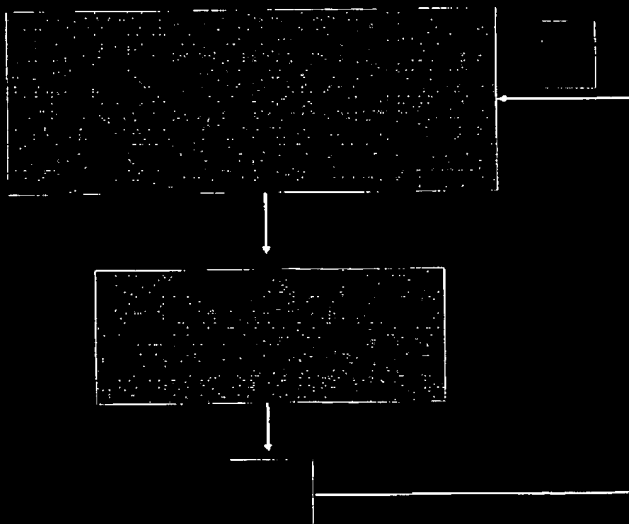
$$\hat{\sigma}^2 = \frac{1}{N} \sum_{n=1}^N \hat{X}_n^H \hat{X}_n$$

- The error in the channel estimate is given by

$$\hat{e} = \hat{H} - H = \frac{1}{N} \sum_{n=1}^N \hat{X}_n \hat{X}_n^H - H$$

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## Overview of EM



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## ML versus EM Channel Estimation

ML

$$\hat{\theta}_{ML} = \underset{\theta}{\operatorname{argmax}} \prod_{i=1}^N p(y_i | \theta)$$

Complexity

EM (iterative)

1. Expectation

$$Q(\theta | \theta^{(t)}) = \sum_{i=1}^N \sum_{k=1}^K \gamma_{ik}(\theta^{(t)}) \log p(y_i | \theta)$$

2. Maximization

$$\theta^{(t+1)} = \underset{\theta}{\operatorname{argmax}} Q(\theta | \theta^{(t)})$$

3. Go back to 1

Complexity

## Block EM Channel Estimation

1. Compute

$$\hat{\theta}_{EM} = \underset{\theta}{\operatorname{argmax}} \sum_{i=1}^N \sum_{k=1}^K \gamma_{ik}(\theta) \log p(y_i | \theta)$$

using

$$\gamma_{ik}(\theta) = \frac{p(y_i | \theta, k)}{\sum_{j=1}^K p(y_i | \theta, j)}$$

2. Compute  $\hat{\theta}_{EM}$  in a similar fashion.

3. Channel and noise variance estimates are given by

$$\hat{\theta}_{EM} = \underset{\theta}{\operatorname{argmax}} \sum_{i=1}^N \sum_{k=1}^K \gamma_{ik}(\theta) \log p(y_i | \theta)$$

## Advantages of EM

- Takes advantage of finite alphabet property of transmit signal.
- Increases likelihood at every iteration and guaranteed to converge.
- Provides MMSE estimate of transmit data.

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## Let's take a trip to Japan

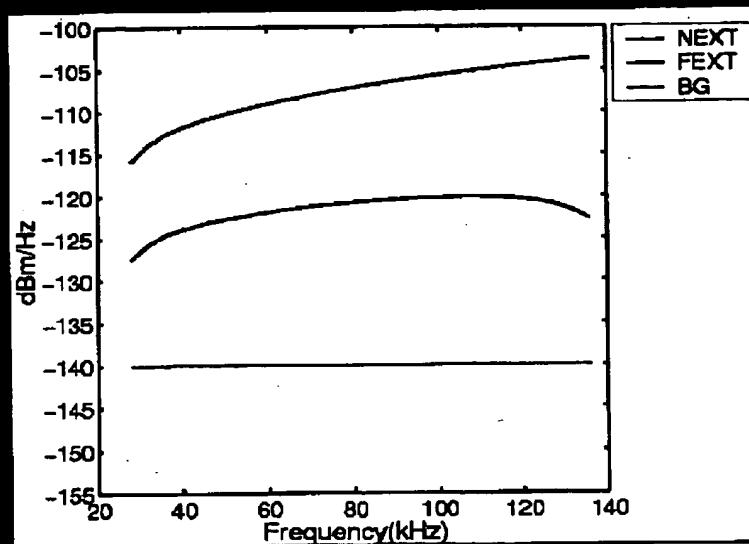


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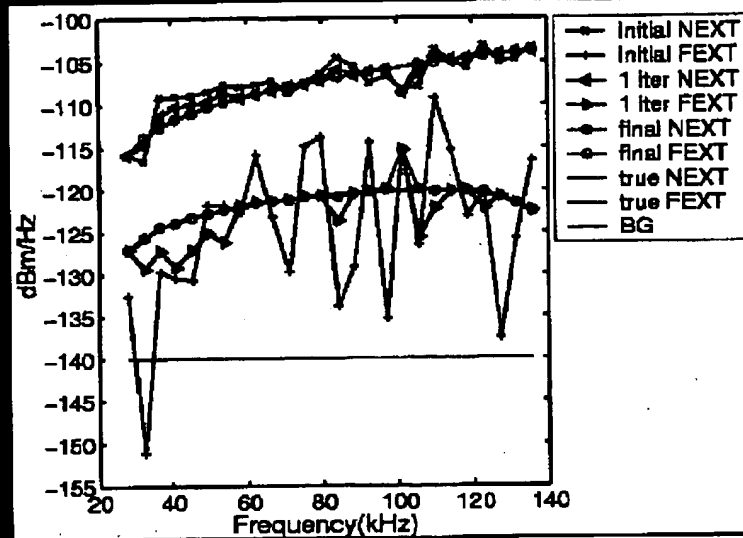
## Simulation Setup

- ADSL-DBM modem
  - 1 NEXT (SSDSL) and
  - 1 FEXT (ADSL)
- 500 m line FEXT source
- Initial condition acquired from previous block
- 10 ms of data ( $L_{tr}=0$ ,  $L=40$ )

## EM Simulation Results



## EM Simulation Results



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### Notation

- Eliminates delay
- Reduces storage
- Track time-variant parameters in an adaptive manner
- Block stationary assumption no longer needed

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## MED Resolutive EM

1. Find  $\frac{dy}{dx}$  and

## 2. Compute

### 3. Update

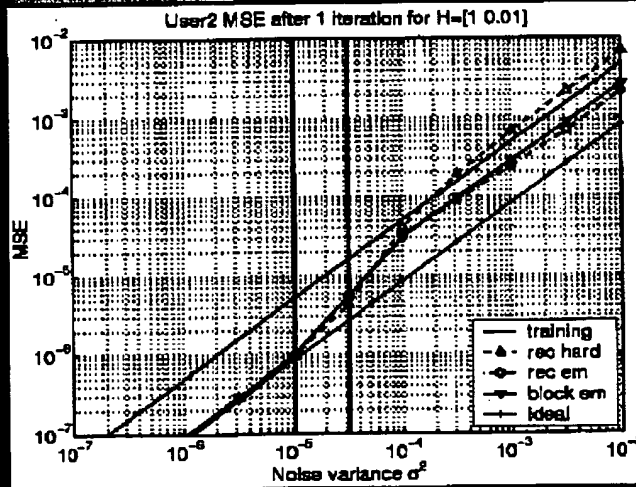
100% **REPUTATION**

## Complexity

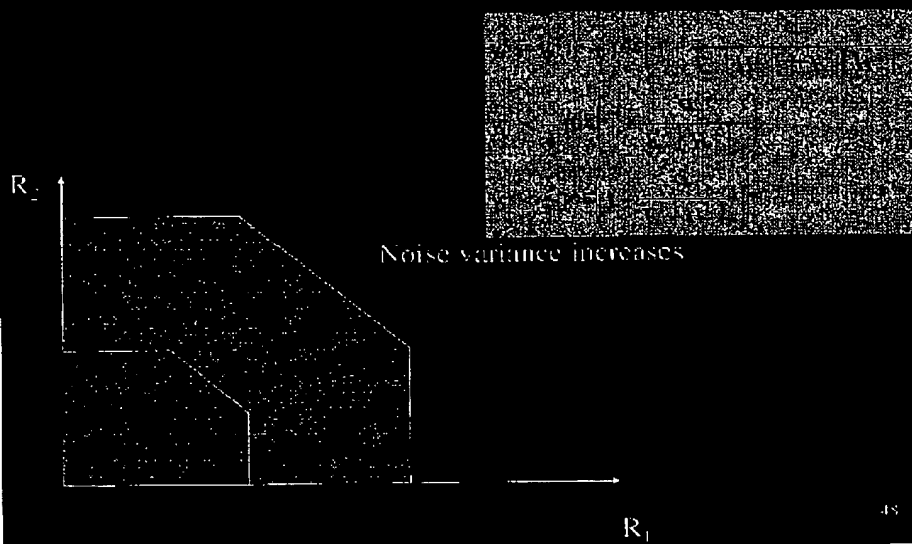
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On average, EM helps for an SNR

Figure 11.11

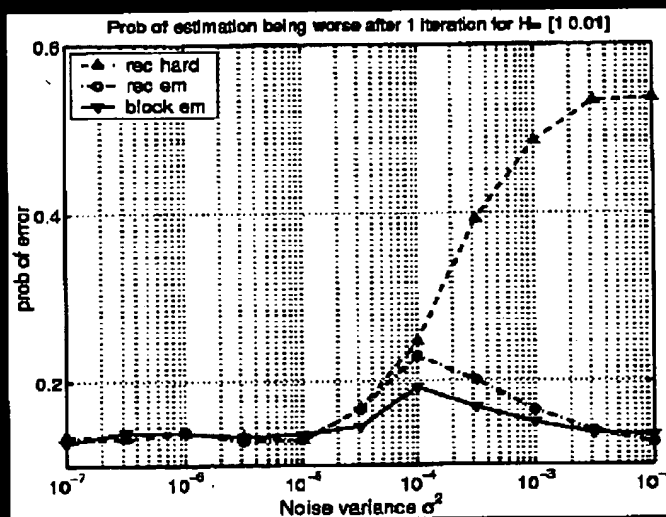


Gaussian MAC Region for  $K=2$





Probability that EM makes MSE worse than blind and training



### Error Floor Analysis for EM w/ Training

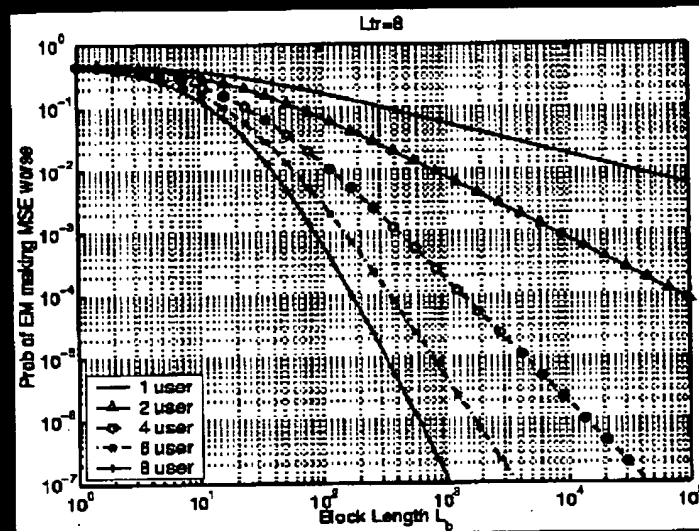
If each user has equal energy  $E_u$ , and we assume that

- $\mathbf{H}_0$  and  $\mathbf{H}_1$  are independent
- Training portion and training+blind are independent then:

$$P_{\text{error}} = \frac{1}{2} \left( 1 - \frac{1}{\sqrt{1 + \frac{E_u}{N_0}}} \right)$$

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## Error Floor for Different K



## Comparison with Modified LMS (MLMS)

### Multiuser Channel ID

$$\hat{\mathbf{H}} = \mathbf{E}[\mathbf{Y} \mathbf{X}^H] \mathbf{E}[\mathbf{X} \mathbf{X}^H]^{-1}$$

$$\hat{\mathbf{H}} = \mathbf{E}[\mathbf{Y} \mathbf{X}^H] \mathbf{E}[\mathbf{X} \mathbf{X}^H]^{-1}$$

#### Advantages of MLMS over EM:

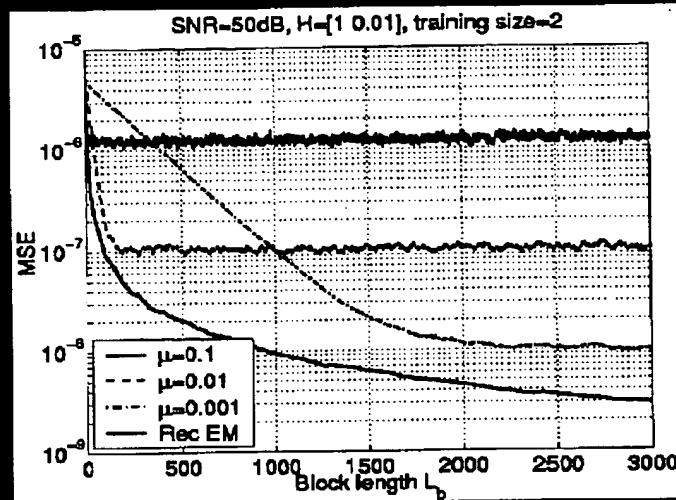
No need to compute second order statistics.

Reduces complexity from  $O(N^2)$  to  $O(N)$

#### Disadvantages

Convergence speed, misadjustment

## MLMS versus Recursive EM inside MAC



## Summary of Contributions

- Identified crosstalk spectrum using accurate and low complexity algorithm
  - Higher data rates
  - 4s Profile 5ms
- Obtained multiuser ML channel and noise estimates
  - Soft decisions better than hard decisions
  - Improved training estimates outside the MAC region
- Developed recursive solution
  - Less storage and delay
- Additional work
  - Extension to MIMO systems
  - Application to coded systems

## Papers

- Aldana, Carvalho, Cioffi, "Channel Estimation for Multicarrier Multiple Input, Single Output Systems using the EM Algorithm", to be submitted to Trans Signal Processing
- Aldana, Cioffi "Channel Tracking for Multiple Input, Single Output Systems using the EM Algorithm", ICC 2001.
- Aldana, Salvekar, Tellado, Cioffi, "MAP Noise Profile Matching for Multicarrier Systems", ICT 2001.
- Aldana, Salvekar, Tellado, Cioffi, "Accurate Noise Estimates in Multicarrier Systems", Fall VTC 2000.
- Salvekar, Aldana, Carvalho, Cioffi, "Crosstalk Profile Detection for use in Multiuser Detection", ICC 2001.
- Zeng, Aldana, Salvekar, Cioffi, "Crosstalk Identification in xDSL systems", IEEE Journal on Selected Areas of Communications.
- Salvekar, Aldana, Tellado, Cioffi, "Peak-to-Average Power Ratio Reduction for Block Transmission Systems in the Presence of Transmitt Filtering", ICC 2001.
- Salvekar, Aldana, Tellado, Cioffi, "Channel Gain Change Detection and Channel Profile Selection in a Multicarrier System", Globecom 99.

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## Acknowledgment

- Advisor: Prof. Cioffi
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- PhD Oral Committee: Prof. Tobagi and Prof. Gill
- Joice
- Family
- Wonderful Friends

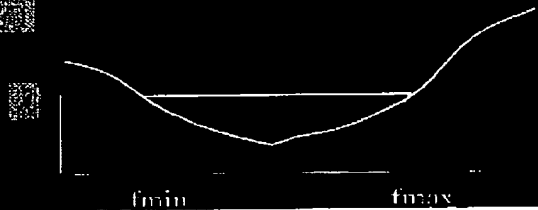
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## Transmit Optimization-Waterfilling

If  $\frac{P(f)}{\sigma^2(f)} > \lambda$ , then pour energy in trough using

$$P(f) = \left( \lambda - \frac{\sigma^2(f)}{P(f)} \right)^+ \quad \text{where } \lambda \text{ is chosen so that}$$

$$\int \left( \lambda - \frac{\sigma^2(f)}{P(f)} \right)^+ df = E$$



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## Crosstalk PSD Formula

$$PSD_{NEXT}(f) = K_{NEXT} PSD_{TX}(f)^* f^{1.5}$$

$$PSD_{NEXT}(f) = K_{NEXT} PSD_{TX}(f)^* |H(f)|^2 f^2 d$$

f: frequency

d: distance

H(f): crosstalk coupling function

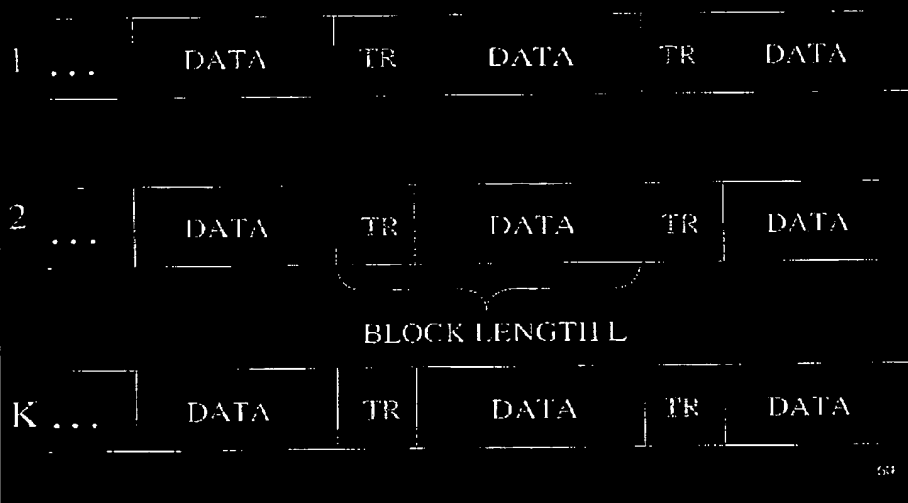
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# FSAN Model for NEXT sources

$$Kf^{\frac{2}{2}} \left( N_1 P_1(f)^{\frac{1}{0.6}} + N_2 P_2(f)^{\frac{1}{0.6}} \right)^{0.6}$$

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## Frame Structure for users 1 to K



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## System Model

A block of received data of length  $L$  can be collected to form:

$$\begin{bmatrix} Y^1 \\ Y^2 \\ \vdots \\ Y^L \end{bmatrix} = \begin{bmatrix} X_1^1 X_2^1 \cdots X_K^1 \\ X_1^2 X_2^2 \cdots X_K^2 \\ \vdots \\ X_1^L X_2^L \cdots X_K^L \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_K \end{bmatrix} + \begin{bmatrix} N^1 \\ N^2 \\ \vdots \\ N^L \end{bmatrix}$$

$$Y = XH + N$$

Problem: Find  $H$  and  $\phi$ , but  $X$  is unknown!! <sup>91</sup>

## System Model

A block of received data of length  $L$  can be collected to form:

$$\begin{bmatrix} Y^1 \\ Y^2 \\ \vdots \\ Y^L \end{bmatrix} = \begin{bmatrix} X_1^1 X_2^1 \cdots X_K^1 \\ X_1^2 X_2^2 \cdots X_K^2 \\ \vdots \\ X_1^L X_2^L \cdots X_K^L \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_K \end{bmatrix} + \begin{bmatrix} N^1 \\ N^2 \\ \vdots \\ N^L \end{bmatrix}$$

$$Y = XH + N$$

$$E\{N^i N^j\} = 0, \quad i \neq j$$

Problem: Find  $H$  and  $\phi$ , but  $X$  is unknown!! <sup>92</sup>

## Review of Talwar Work

### ■ ILSP Algorithm

Given  $H$

Find LS estimate of  $X$  (since you have enough diversity)

Project onto  $X_{\text{hat}}$

Using this  $X_{\text{hat}}$  find LS estimate of  $H$  and repeat.

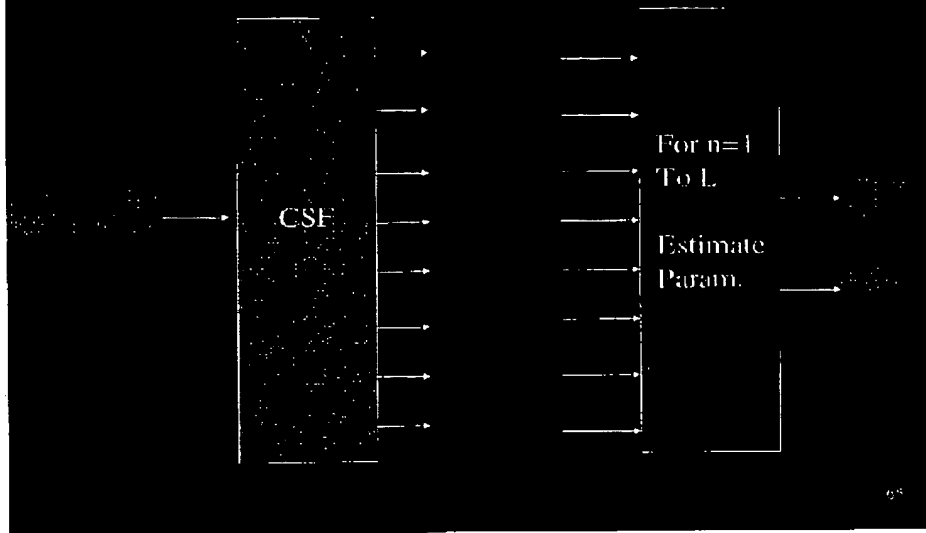
## EM Initial Condition for BPSK

$$\min_{\Delta} \max_{\Delta} \left\| Y - H \Delta \right\|_2^2$$

$$\begin{aligned} \Delta_1 &= H_1^T H_1^{-1} Y \\ \Delta_2 &= H_2^T H_2^{-1} Y \\ \text{For } \Delta_1 \text{ and } \Delta_2 \neq 0 \\ \min_{\Delta} \left\| \sum_{i=1}^2 \Delta_i H_i^T H_i \Delta_i \right\|_2^2 &= \frac{Y^T Y}{\sum_{i=1}^2 H_i^T H_i} \\ \Delta_1 &= \frac{Y}{\sum_{i=1}^2 H_i^T H_i} \end{aligned}$$



## Block Diagram of EM



## Explanation of Recursive Formula

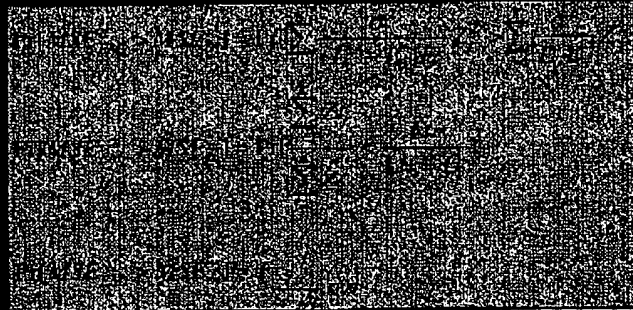
$$\begin{aligned}
 \mu_1 &= \sum_{i=1}^N x_i / N \\
 \mu_2 &= \sum_{i=1}^N x_i^2 / N \\
 \mu_3 &= \sum_{i=1}^N x_i^3 / N \\
 \mu_4 &= \sum_{i=1}^N x_i^4 / N \\
 \mu_5 &= \sum_{i=1}^N x_i^5 / N \\
 \mu_6 &= \sum_{i=1}^N x_i^6 / N \\
 \mu_7 &= \sum_{i=1}^N x_i^7 / N \\
 \mu_8 &= \sum_{i=1}^N x_i^8 / N \\
 \mu_9 &= \sum_{i=1}^N x_i^9 / N \\
 \mu_{10} &= \sum_{i=1}^N x_i^{10} / N \\
 \mu_{11} &= \sum_{i=1}^N x_i^{11} / N \\
 \mu_{12} &= \sum_{i=1}^N x_i^{12} / N \\
 \mu_{13} &= \sum_{i=1}^N x_i^{13} / N \\
 \mu_{14} &= \sum_{i=1}^N x_i^{14} / N \\
 \mu_{15} &= \sum_{i=1}^N x_i^{15} / N \\
 \mu_{16} &= \sum_{i=1}^N x_i^{16} / N \\
 \mu_{17} &= \sum_{i=1}^N x_i^{17} / N \\
 \mu_{18} &= \sum_{i=1}^N x_i^{18} / N \\
 \mu_{19} &= \sum_{i=1}^N x_i^{19} / N \\
 \mu_{20} &= \sum_{i=1}^N x_i^{20} / N
 \end{aligned}$$

## Explanation for F-distribution

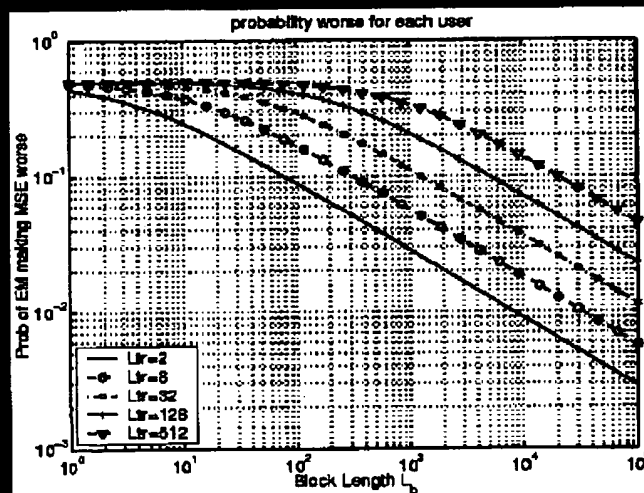
In each user dimension

$$\sigma^2 = \frac{1}{L} \sum_{l=1}^L \sigma_l^2$$

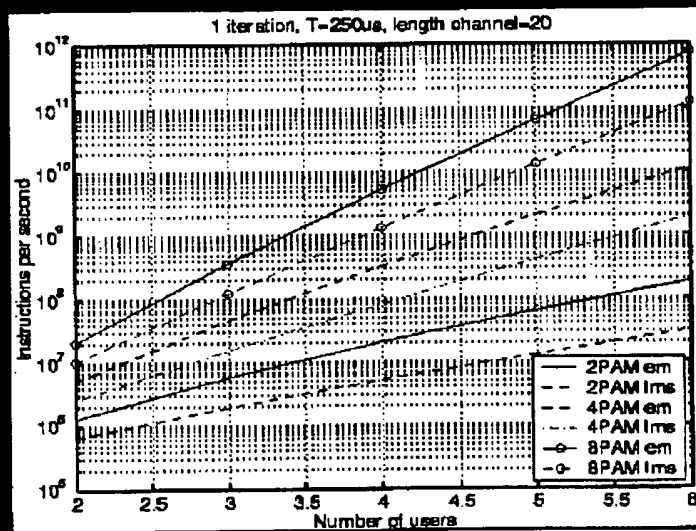
$$\sigma^2 = \frac{1}{L} \sum_{l=1}^L \sigma_l^2 = \frac{1}{L} \sum_{l=1}^L \sigma_l^2$$



## MSE Channel behavior for Lr varying



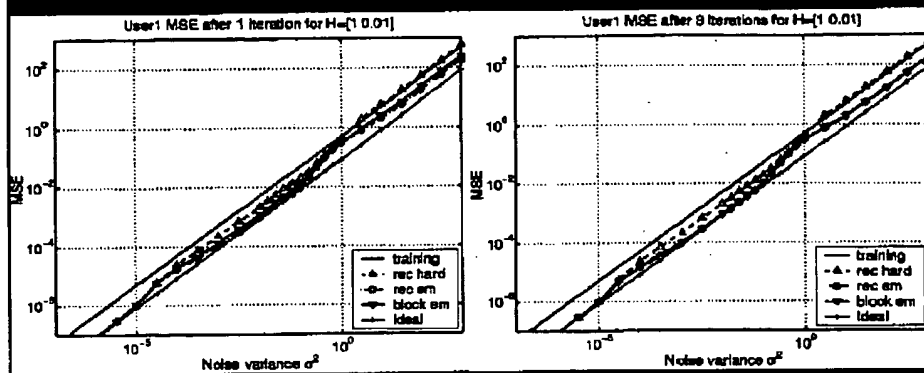
## MIPS Plot for EM and LMS



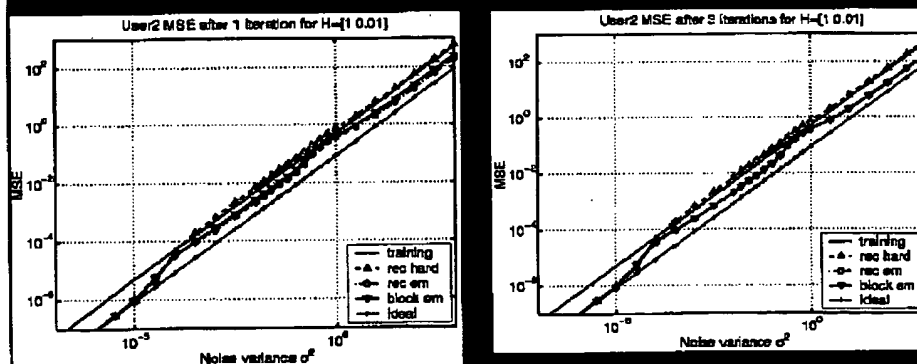
## Gaussian MAC Capacity

$$\begin{aligned} \text{Gaussian MAC Capacity} &= \frac{1}{\log_2} \left( \frac{\sum_{k=1}^K \left( \frac{P_k}{\sigma^2} \right)}{\sum_{k=1}^K \left( \frac{P_k}{\sigma^2} \right) + 1} \right) \\ &= \frac{1}{\log_2} \left( \frac{\sum_{k=1}^K \left( \frac{P_k}{\sigma^2} \right)}{\sum_{k=1}^K \left( \frac{P_k}{\sigma^2} \right) + 1} \right) \\ &= \frac{1}{\log_2} \left( \frac{\sum_{k=1}^K \left( \frac{P_k}{\sigma^2} \right)}{\sum_{k=1}^K \left( \frac{P_k}{\sigma^2} \right) + 1} \right) \end{aligned}$$

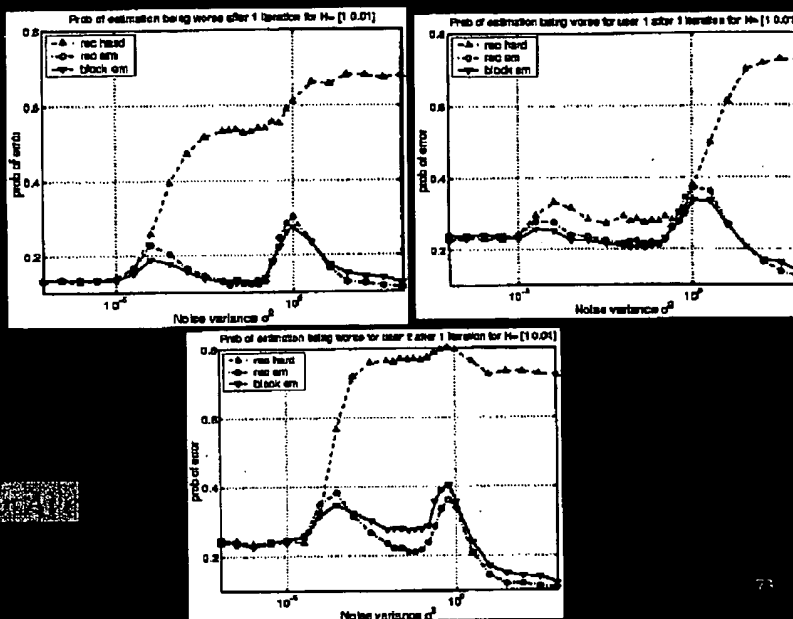
## User 1 MSE



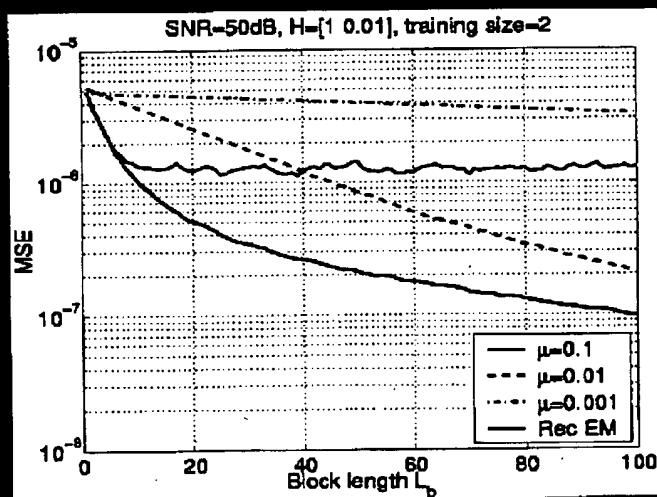
## User 2 MSE

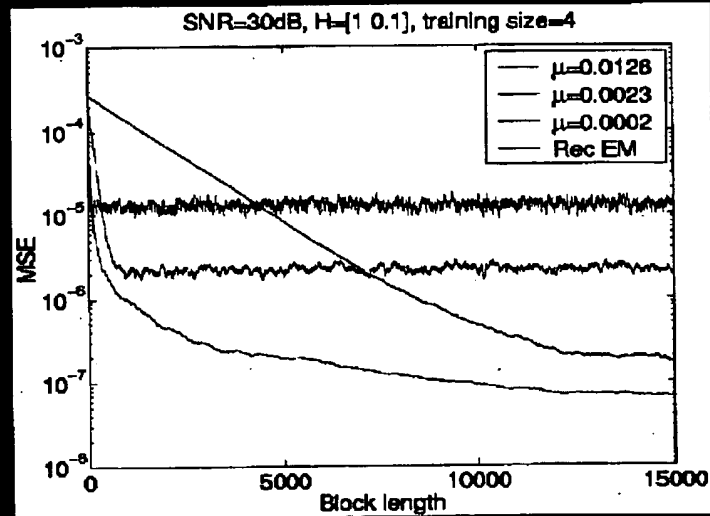
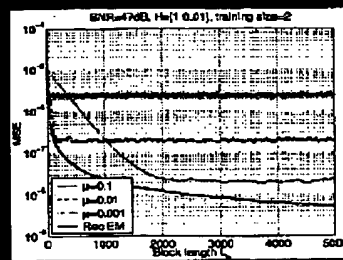
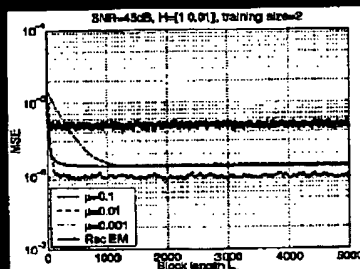
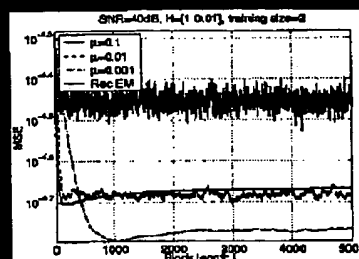
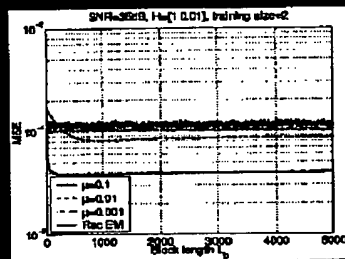


Error

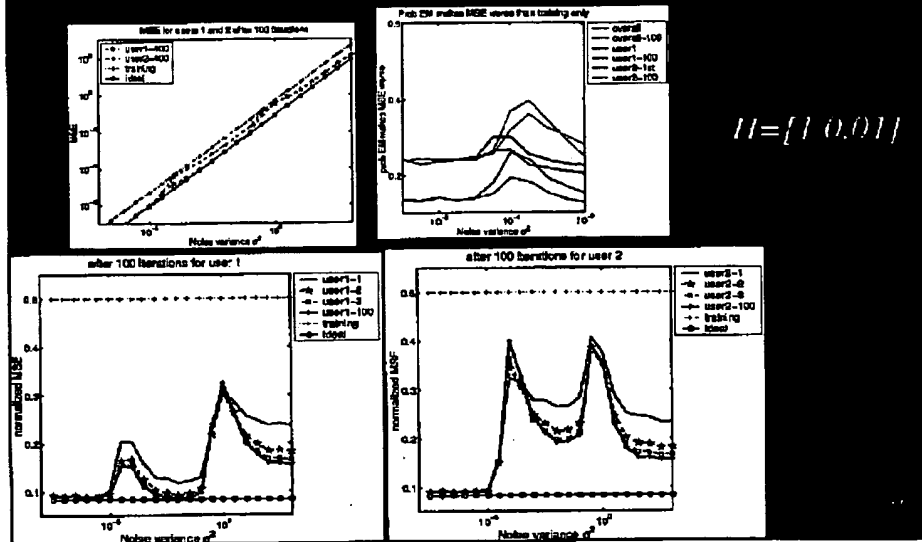


Zoomed LMS versus EM

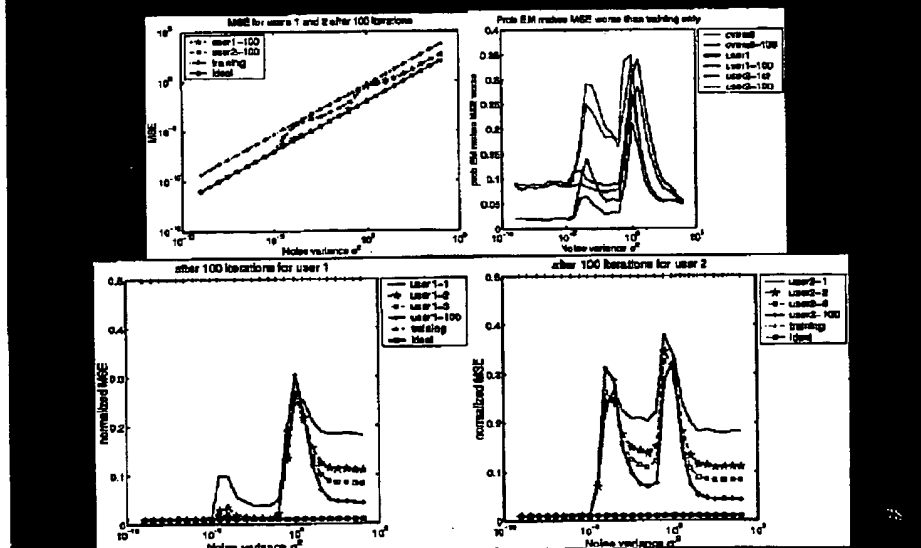


LMS versus EM for  $H=[1 \ 0.1]$  $H=[1 \ 0.01]$ 

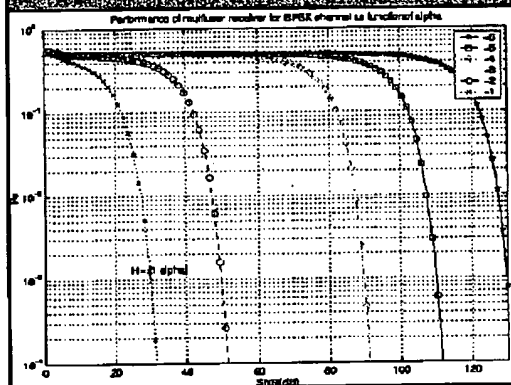
## Block Recursive EM for $L_{tr}=2$ , $L_b=10$



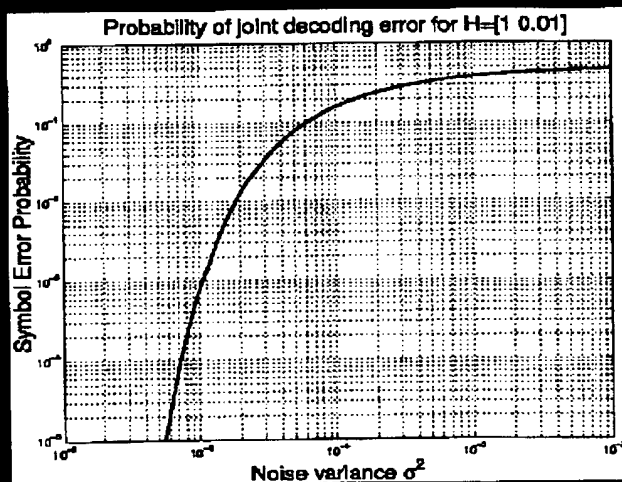
## Block Recursive EM for $L_{tr}=2$ , $L_b=100$



## 2 user BPSK performance



## Psymbol error for H=[1 0.01]





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